Centro de Investigação em Matemática e Aplicações
Departamento de Matemática
Programa de Doutoramento em Matemática

Seminário (online)

11 de maio de 2022, 15h

A brief survey of the vector case of the Calculus of Variations

Luís Bandeira

Departamento de Matemática, CIMA, Escola de Ciências e Tecnologia, Universidade de Évora

Consider the typical variational problem

$$\text{Minimize in } u \in A: \quad I(u) = \int_\Omega \varphi\big(\nabla u(x)\big)dx$$

where feasible fields $u \in A$ comply with boundary conditions (for instance)

$$u: \Omega \subset \mathbb{R}^N \to \mathbb{R}^m,$$

A is a certain subset of typical Sobolev spaces, and the integrand

$$\varphi: \mathbb{R}^{m\times N} \to \mathbb{R}$$

is assumed to be continuous. These vector variational problems are fundamental in applications to models in non-linear elasticity of solids. It is well-known that a fundamental property to show existence of states of minimum energy is the so-called weak lower semicontinuity of the energy functional I, and this in turn is equivalent to the quasiconvexity property of the integrand φ. This property for φ is extremely difficult to check in specific examples (if not impossible) so that other important convexity properties were sought. These are polyconvexity (sufficient condition for quasiconvexity), and rank-one convexity (necessary condition for quasiconvexity).

After a general survey, we report our own work about the analysis of the situation concerning the equivalence of quasiconvexity and rank-one convexity of integrands φ, for the case $m = 2$ ($N > 1$), which is an open problem.

UNIVERSIDADE DE ÉVORA
ESCOLA DE CIÊNCIAS E TECNOLOGIA
DEPARTAMENTO DE MATEMÁTICA

CIMA Centro de Investigação em Matemática e Aplicações
UNIVERSIDADE de ÉVORA · UNIVERSIDADE da MADEIRA · ISEL
UIDB/04674/2020

FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

REPÚBLICA PORTUGUESA
XXII GOVERNO