The Heuristic Approach to movement optimization on single-track part of the railway net

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Abstract: This paper represents our solution for the problem of movement organization based on timetable optimization on the problematic part of railway system, i.e. single-track line. The approximate solution of this problem was founded on the heuristic method. The method gives the exact results in the case of limited amount of parameters and also can be used in the case with huge number of parameters due to reasonable computational time.

Key words: scheduling theory, algorithm, single-track railway problem.

Introduction

Single track railways are of great interest to scheduling theory because nowadays they are the most weak chain in the railroad transportation all over the world and especially in Russia. The article tend to solving bottleneck problems in transport network. Bottleneck is a part of the way with low bandwidth compare with other parts of the same road. Often it is the railway line with a limited number of tracks. It can also be a narrow bridge, tunnel or narrow causeway. Presence of such railway parts often cause delays and Timetable failure, and it is necessary to obtain good solution by optimization of schedule (Timetable). This task with single line track is known to be NP-hard; we made an attempt to create an Heuristic algorithm which will help to minimize reasonable computational time for solving this problem.

Problem formulation

We consider that arrival numerous applications to stations 1 and 2 are known in advance. Considering the trains with following parameters:
\[ N = N_1 \cup N_2 \] — set of trains;
\[ N_1 = \{1, 2, \ldots, n\} \] — set of trains arrived at the station 1;
\[ N_1 = \{1, 2, \ldots, m\} \] — set of trains arrived at the station 2;
\[ r^i_1 \] — planning time of \( i \) train to station 1;
\[ r^j_2 \] — planning time of \( j \) train to station 2;
\[ d^i_1 \] — the due date of arrival \( i \) train \( i \in (1, n) \) at the station 1 to the station 2;
\[ d^j_2 \] — the due date of arrival \( j \) train \( j \in (1, m) \) at the station 2 to the station 1;
\[ p \] — the average time of movement the train \((p = \text{const})\);
\[ \delta \] — headway between trains.

**Fact data**

\[ C^i_1 \] — real arrival time of \( i \) train to station 2;
\[ C^j_2 \] — real arrival time of \( j \) train to station 1;
\[ S^i_1 \] — real departure time of the \( i \) train;
\[ S^j_2 \] — real departure time of the \( j \) train.

The objective function is following:
\[ T_{\Sigma}(\pi) = \sum_{i=1}^{N} \max\{0, C_i - d_i\} \]

Our task was to create the train schedule optimizing the total delay of movement trains on the single-track part of the railway net. (Fig.1)

![Bottleneck problem](image)

**Note 1.** If \( \delta = p \), i.e., on the railway at the time can be no more than one of train. The problem reduces to \((m + n)\) service requirements
on a single device to the agreed arrival time and the due date. There are number of polynomial algorithms for solving such problems [1-3].

Note 2. Further, we assume that $\delta = 0$, i.e. the delay time of each train is negligible compared to the distance between stations.

The heuristic algorithm

The algorithm consists of two parts: direct flow and indirect flow.

Definition 1. Batch$(n, i)$ — a set, containing $n$ trains, departed from the station 2 at the same moment $C_n^i = \max\{r_n^i, r_1^i + p\}$.

Under the direct flow we create Batch$(n, i)$ (see def.1). For Batch$(n, i)$ we determine all possible departure moments $S_n^i$. In this way the trains from the station 1 with $r_1^i \in (S_n^i - p; S_n^i + p)$ will depart at the moment $t = S_n^i + p$. At this stage we analyze the getting results of objective function $F(S_n^i)$; and afterwards we choose the smallest meaning. Thus we got the optimal departure time $S_n$ for the least train. Similarly we find the optimal departure time $S_{n-1}, S_{n-2}, ... S_1$.

Definition 2. Batch$(m, j)$ — a set, containing $m$ trains, departed from the station 1 at the same moment $S_m^j = \max\{r_m^j, r_1^j + p\}, j \in (1, n)$.

In the calculation of the indirect flow we create Batch$(m, j)$.

The development of this algorithm allows to simplify the task of schedule optimization and it is especially use for failure movement.

REFERENCES