Zonal control of lumped systems on different classes of feedback functions

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In this work we investigate a class of feedback control problems for dynamic, in the general case, nonlinear objects involving lumped parameters. For synthesized control actions we introduce the notion of zonality that means constancy of the synthesized control parameters’ values in each of the subsets (zones). These subsets are obtained by partitioning the set of all possible states of the object investigated. Formulas for the gradient of the target functional with respect to the optimizable parameters of the synthesized controls are obtained. These formulas can be used to build numerical solution schemes on basis of first order iterative optimization methods. Results of numerical experiments obtained by solving some test problems are given.

Let the controlled process be described by the following nonlinear differential equations system:

\[ \dot{x}(t) = f(x(t), u(t), p), \quad t > 0, \quad (1) \]
\[ x(0) = x^0 \in X^0 \subset \mathbb{R}^n, \quad p \in P \subset \mathbb{R}^m, \quad (2) \]

where \( x(t) \) is the \( n \)-dimensional vector function of the process state; \( u(t) \in U \) is the \( r \)-dimensional control vector function; \( U \subset \mathbb{R}^r \) is the closed set of the control actions’ admissible values; \( p \) is the \( m \)-dimensional vector of the process’s constant parameters, the values of which are uncertain, but there is a set of their possible values \( P \) and the density (weighting) function \( \rho_P(p) \geq 0 \) defined on \( P \); \( X^0 \) is the set of possible values of the process’s initial states with the density (weighting) function \( \rho_{X^0}(x^0) \geq 0 \) given.

Control of the process (1) is realized with the use of feedback; the state vector \( x(t) \) may be measured fully or partially. Observations of the process state may be carried out at discrete points of time or continuously. To control the process, we propose to choose the values of the synthesized control actions according to a subset (zone) the measured current process...
The subsets are obtained by partitioning the set of all possible phase states of the object.

The objective of the feedback control problem considered is to determine the values of the parameters of the zonal control actions \( u(t) \) minimizing the following functional

\[
J(u) = \int_{X^0} \int_P I(u, T; x^0, p) \rho_{X^0}(x^0) \rho_P(p) dP dX^0 / (\text{mes} X^0 \cdot \text{mes} P), \tag{3}
\]

\[
I(u, T; x^0, p) = \int_0^T g(x(t), u(t)) dt + \Phi(x(T), T). \tag{4}
\]

Here \( x(t) = x(t; x^0, p, u) \) is the solution to the system (1) under the admissible control \( u(t) \), initial state \( x^0 \), and the values of the parameters \( p; T = T(x^0, P) \) is the corresponding completion time of the process, which can be either a fixed quantity \( T = T(x^0, P) = \text{const} = T \), or an optimizable function of the values of the initial state and of the object’s parameters \( T = \{ T(x^0, P) : T(x^0, P) \leq \bar{T}, x^0 \in X^0, p \in P \} \), where \( \bar{T} \) is given. The latter case arises in, as a rule, speed-in-action problems for control systems. We considered both cases.

The functional (3) and (4) defines the quality of control which is optimal on the average with respect to the admissible values \( x^0 \in X^0 \) and \( p \in P \). Denote by \( X \subset \mathbb{R}^n \) the set of all possible states of the object under different admissible initial states \( x^0 \in X^0 \), the values of the parameters \( p \in P \), and the controls \( u(t) \in U \) for \( t \in [0, \bar{T}] \).

Let the set \( X \) be partitioned into given number \( L \) of open subsets \( X^i \) such that

\[
\bigcup_{i=1}^L X^i = X, \quad X^i \cap X^j = \emptyset, \quad i \neq j, \quad i, j = 1, 2, ..., L,
\]

where \( \overline{X^i} \) is the closure of the set \( X^i \). In the work we consider the following four types of feedback control problems, which differ in organization of feedback with the object and, therefore, in formation of the control actions’ values.

**Problem 1.** There are points of time \( \tau_j \in [0, \bar{T}], \quad j = 0, 1, ..., N, \tau_0 = 0 \) given, at which it is possible to observe the current state of the process \( x(\tau_j) \in X \). The frequency of these observations is such that when
the process state belongs to some subset, it is observed at least once. The values of the control \( u(t) \), which are constant for \( t \in [\tau_j, \tau_{j+1}) \), are assigned depending on the value of the last observed current process state, namely, depending on the subset \( X^i \) with \( i = 1, 2, ..., L \) of the phase space \( X \) which the current state belongs to. Therefore

\[
u(t) = v^i = \text{const}, \quad x(\tau_j) \in X^i, \quad t \in [\tau_j, \tau_{j+1}),
\]

\[v^i \in U \subset R^r, \quad i = 1, 2, ..., L, \quad j = 0, 1, ..., N - 1, \quad \tau_N = \bar{T}.
\]

It is required to determine zonal values of the control \( v^i \), \( i = 1, 2, ..., L \), optimizing the functional (3).

**Problem 2.** The control actions are defined in the form of a linear function of the results of observations of the state variables at given discrete points of time \( \tau_i \in [0, \bar{T}] \), \( i = 0, 1, ..., N \):

\[
u(t) = K_1^i \cdot x(\tau_j) + K_2^i, \quad t \in [\tau_j, \tau_{j+1}), \quad x(\tau_j) \in X^i, \quad t \in [0, \bar{T}],
\]

\[i = 1, 2, ..., L, \quad j = 0, 1, ..., N - 1.
\]

Here \( K_1^i \) is the \((r \times n)\) matrix and \( K_2^i \) is the \( r\)-dimensional vector which are constant for \( t \in [\tau_{j-1}, \tau_j) \). The problem is to determine the values \( K_1^1, K_2^1, K_1^2, K_2^2, ..., K_1^L, K_2^L \), optimizing the functional (3).

**Problem 3.** Continuous observation of the process state is carried out; the control actions take zonal values of the control:

\[
u(t) = w^i = \text{const}, \quad x(t) \in X^i, \quad t \in [0, \bar{T}],
\]

\[w^i \in U \subset R^r, \quad i = 1, 2, ..., L.
\]

It is required to determine the zonal values of the control \( w^i \), \( i = 1, 2, ..., L \), optimizing the functional (3).

**Problem 4.** Continuous observation of the process state is carried out; the control actions are defined by a linear function of the measured current values of the process variables:

\[
u(t) = L_1^i \cdot x(t) + L_2^i, \quad x(t) \in X^i, \quad t \in [0, \bar{T}], i = 1, 2, ..., L, \quad j = 0, 1, ..., N - 1.
\]
Here $L_1^i$ is the $(r \times n)$ matrix and $L_2^i$ is the $r$-dimensional vector which are constant for each subset $X^i$, i.e. while $x(t) \in X^i$. It is required to determine the values $L_1^i, L_2^i, i = 1, 2, ..., L$, optimizing the functional (3). Note that in all the four problems, the synthesized controls are defined by the finite-dimensional constant vectors and matrices.

To solve the optimization problems stated above numerically and to determine the control actions from the classes (5)-(8), we propose to use first order optimization methods and the corresponding standard software [1]. For this purpose, we obtained formulas for the gradient of the target functional using the technique of the target functional increment obtained at the expense of the optimizable arguments increment [2]. Results of numerical experiments carried out by the example of the solution to several model problems are given.

REFERENCES