A Conjugate Gradient Projection Algorithm for systems of Large-Scale Nonlinear Monotone Equations

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Abstract

Systems of nonlinear equations generally are a family of problems that is so close to optimization problems and often arise in the applied sciences, technology and industry. In general, the system of nonlinear equations can be formulated mathematically by

\[ G(x) = 0, \quad \text{subject to } x \in \mathbb{R}^n, \quad (1) \]

where \( G : \mathbb{R}^n \to \mathbb{R}^n \) is a continuous function. In particular, the nonlinear monotone equations are a class of nonlinear equations whenever \( G(x) \) satisfies the following monotonicity condition

\[ (G(x) - G(y))^T (x - y) \geq 0, \quad \text{for all } x, y \in \mathbb{R}^n, \]

guaranteeing that the solution set of (1) is a convex set.

We propose two derivative-free approaches for solving a large-scale nonlinear monotone system. The framework firstly generates a specific direction then employs a line search to construct a new point. If the new point doesn’t solve the problem, the projection technique constructs an appropriate hyperplane that separates the current iterate from the solutions of the problem. Then the projection of the new point onto the hyperplane will determine the next iterate. Thanks to the low memory requirement, we use two new conjugate gradient directions. The global convergence is established Under appropriate conditions.