The authors study the variational problem with affine boundary conditions

\[
(P_v) \quad \min \left\{ \int_\Omega g(\nabla u(x))dx : u - \langle v, \cdot \rangle \in W_0^{1,p}(\Omega) \right\},
\]

where \( g : \mathbb{R}^m \to \mathbb{R} \cup \{\infty\} \) is a lower semicontinuous proper function, and its relaxed version

\[
(\mathcal{RP}_v) \quad \min \left\{ \int_\Omega g^{**}(\nabla u(x))dx : u - \langle v, \cdot \rangle \in W_0^{1,p}(\Omega) \right\}
\]

where \( g^{**} \) is the bipolar function of \( g \).

The authors define a minimal and a maximal solution of the convexified problem \((\mathcal{RP}_v)\) and show that these solutions depend Lipschitz continuously on \( v \). These are used to show density results for subsets of \( S^{**} \) given by

\[
S^{**} = \{ \rho \in C(\mathbb{R}^n, C_0(\Omega)) : x \mapsto \langle v, x \rangle + \rho(v)x \text{ is a solution of } (\mathcal{RP}_v) \text{ for all } v \in \text{dom}(g^{**}), \\
\rho \equiv 0 \text{ if } v \notin \text{dom}(g^{**}) \}
\]

Using these density results and a Baire category argument, it is then shown that in the set of \( v \) where \((P_v)\) admits a minimizer \( \hat{s}(v)(\cdot) \), this minimizer can be chosen Lipschitz continuous with Lipschitz constant \( \|\Omega\| = \max_{x \in \Omega} |x| \). The minimizer can be chosen arbitrarily close to a \( \|\Omega\| \)-Lipschitz continuous selection \( \tilde{s}(v) \) of the relaxed problem \((\mathcal{RP}_v)\) which always remains above the affine function.

Reviewed by Matthias Kurzke

References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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