Existence and location result for a fourth order boundary value problem. (English summary)


In this paper the authors prove the existence of a solution using the upper and lower solution method for the fourth-order fully nonlinear equation

\[ u^{iv} = f(t, u, u', u'', u'''), \quad 0 < t < 1, \]

with Lidstone boundary conditions

\[ u(0) = u''(0) = u(1) = u''(1) = 0, \]

where \( f: [0, 1] \times \mathbb{R}^4 \rightarrow \mathbb{R} \) is a continuous function. They prove that the solution and its first two derivatives lie between the lower and upper solutions and its first and second derivatives, respectively. Precisely, they prove that there exists at least one solution \( u(t) \in C^4([0, 1]) \) such that \( \alpha(t) \leq u(t) \leq \beta(t), \alpha'(t) \leq u'(t) \leq \beta'(t), \alpha''(t) \leq u''(t) \leq \beta''(t) \), for \( t \in [0, 1] \). Here \( \alpha, \beta \) are lower and upper solutions of (1) and (2). For this purpose they need to assume a Nagumo type growth condition in the third derivative of \( f \). Furthermore, they prove that the

\[ \alpha'(0) - \beta'(0) \leq \min\{\beta(0) - \beta(1), \alpha(1) - \alpha(0), 0\} \]

is a necessary condition for the existence and location of the solution. This is illustrated with a nice counterexample. It is to be noted that equations (1) and (2) model the deformation of an elastic beam.

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