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Kleinian groups and holomorphic dynamics. (English summary)
Dynamical systems and functional equations (Murcia, 2000).

In this article the authors give examples of how the theory of iterated polynomials can be exploited to prove results about subgroups of $\text{PSL}(2, \mathbb{C})$ acting on the Riemann sphere.

The first set of examples involves three specific families of Fuchsian groups, one consisting of the Hecke groups and the other two of groups generated by two parabolic elements. R. Bowen and C. Series [Inst. Hautes Études Sci. Publ. Math. No. 50 (1979), 153–170; MR0556585 (81b:58026)] proved that for each Fuchsian group there exists a map defined on $\mathbb{R} \cup \{\infty\}$ which is orbit equivalent to the group. The authors construct this map for elements of each of their three families; together with the associated Markov matrix. In the case of Hecke groups they use this result to identify the upper bound $\log 2$ for the topological entropy of the corresponding subshift (it is known [D. Lind and B. Marcus, An introduction to symbolic dynamics and coding, Cambridge Univ. Press, Cambridge, 1995; MR1369092 (97a:58050)] that the entropy is equal to the spectral radius of the Markov matrix). For one of the other two families they point out that the Markov matrix is independent of parameters.

The second part of the paper concerns the question of whether a given subgroup of $\text{PSL}(2, \mathbb{C})$ is discrete or not. The authors consider a one-parameter-family of two-generator groups $G = \langle A, B \rangle$, some of which are cusp groups [see L. Keen and C. Series, Topology 32 (1993), no. 4, 719–749; MR1241870 (95g:32030)]. To each such group one can associate two values $\beta$ and $\gamma$ (arising from traces of group elements). The authors show that if the group is Kleinian and $\gamma = -4$, then $\gamma$ lies neither in the interior of the filled Julia set of the quadratic $z \rightarrow z(z - \beta)$, nor in the interior of the filled Julia set of the cubic $z \rightarrow z(1 + \beta - z)^2$. This is based on well-known results about two-generator groups [R. Brooks and J. P. Matelski, in Riemann surfaces and related topics: Proceedings of the 1978 Stony Brook Conference (State Univ. New York, Stony Brook, N.Y., 1978), 65–71, Ann. of Math. Stud., 97, Princeton Univ. Press, Princeton, N.J., 1981; MR0624805 (82j:10045); F. W. Gehring and G. J. Martin, Bull. Amer. Math. Soc. (N.S.) 21 (1989), no. 1, 57–63; MR0974424 (90k:30086)].

Reviewed by Marianne Freiberger

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