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Existence of solutions to differential inclusions and to time optimal control problems in the autonomous case. (English summary)


Let $X$ be a vector space. A set $K \subset X$ is called almost convex if for every $\xi \in \text{co} K$ there exist $\lambda_1$ and $\lambda_2$, $0 \leq \lambda_1 \leq 1 \leq \lambda_2$, such that $\lambda_1 \xi \in K$, $\lambda_2 \xi \in K$. The authors prove the following theorem.

Theorem. Let $\Omega \subset \mathbb{R}^N$ be open, and let $F$, from $\Omega$ to the nonempty subsets of $\mathbb{R}^N$, be upper semicontinuous with bounded, closed, and almost convex values. Then the Cauchy problem $y'(s) \in F(y(s))$, $y(0) = x_0 \in \Omega$, admits a solution defined on some interval $[-\delta, \delta]$, $\delta > 0$. Moreover, for every $\tau \in [-\delta, \delta]$, the attainable set at $\tau$, $A_{x_0}(\tau)$, is closed and coincides with $A_{\text{co} x_0}(\tau)$, the attainable set at $\tau$ of the convexified problem $y'(s) \in \text{co} F(y(s))$, $y(0) = x_0$. Clearly, the main feature of this result is the fact that a weaker assumption of almost convexity is used instead of convexity.

As a corollary, the authors prove the existence of a time optimal solution for the control system $x'(t) = f(x(t), u(t))$, $u(t) \in U(x(t))$, under the assumption that the images of the map $F(x) = f(x, U(x))$ are almost convex instead of the classical Filippov assumption of convexity.

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References


Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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